3.1

3. Devise an algorithm that finds the sum of all the integers in a list.

5. Describe an algorithm that takes as input a list of n integers in nondecreasing order and produces the list of all values that occur more than once. (Recall that a list of integers is nondecreasing if each integer in the list is at least as large as the previous integer in the list.)

7. Describe an algorithm that takes as input a list of n integers and finds the location of the last even integer in the list or returns 0 if there are no even integers in the list.

13. List all the steps used to search for 9 in the sequence 1, 3, 4, 5, 6, 8, 9, 11 using
a) a linear search.
b) a binary search.

56. Use the cashier's algorithm to make change using quarters, dimes, nickels, and pennies for
a) 87 cents.
b) 49 cents.
c) 99 cents.
d) 33 cents.

61. Use Algorithm 7 to schedule the largest number of talks in a lecture hall from a proposed set of talks, if the starting and ending times of the talks are 9:00 A.M. and 9:45 A.M.;
9:30 A.M. and 10:00 A.M.; 9:50 A.M. and 10:15 A.M.;
10:00 A.M. and 10:30 A.M.; 10:10 A.M. and 10:25 A.M.;
10:30 A.M. and 10:55 A.M.; 10:15 A.M. and 10:45 A.M.;
10:30 A.M. and 11:00 A.M.; 10:45 A.M. and 11:30 A.M.;
10:55 A.M. and 11:25 A.M.; 11:00 A.M. and 11:15 A.M.

3.2

In Exercises $1 - 14$, to establish a big-O relationship, find witnesses C and k such that $|f(x)| \leqslant C|g(x)|$ whenever $x > k$.

1. Determine whether each of these functions is $O(x)$.
a) $f(x) = 10$
b) $f(x) = 3x + 7$
c) $f(x) = x^2 + x + 1$
d) $f(x) = 5 \log x$
e) $f(x) = \lfloor x \rfloor$
f) $f(x) = \lceil x / 2 \rceil$

2. Determine whether each of these functions is $O(x^2)$.
a) $f(x) = 17x + 11$
b) $f(x) = x^2 + 1000$
c) $f(x) = x \log x$
d) $f(x) = x^4/2$
e) $f(x) = 2^x$
f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

10. Show that x^3 is $O(x^4)$ but that x^4 is not $O(x^3)$.

12. Show that $x \log x$ is $O(x^2)$ but that x^2 is not $O(x \log x)$.

14. Determine whether x^3 is $O(g(x))$ for each of these functions $g(x)$.
a) $g(x) = x^2$
b) $g(x) = x^3$
c) $g(x) = x^2 + x^3$
d) $g(x) = x^2 + x^4$
e) $g(x) = 3^x$
f) $g(x) = x^3/2$

32. Show that if $f(x)$ and $g(x)$ are functions from the set of real numbers to the set of real numbers, then $f(x)$ is $O(g(x))$ if and only if $g(x)$ is $\Omega(f(x))$.

45. If $f_1(x)$ and $f_2(x)$ are functions from the set of positive integers to the set of positive real numbers and $f_1(x)$ and $f_2(x)$ are both $\Theta(g(x))$, is $(f_1 - f_2)(x)$ also

$\Theta(g(x))$? Either prove that it is or give a counterexample.

46. Show that if $f_1(x)$ and $f_2(x)$ are functions from the set of positive integers to the set of real numbers and $f_1(x)$ is $\Theta(g_1(x))$ and $f_2(x)$ is $\Theta(g_2(x))$, then $(f_1 f_2)(x)$ is $\Theta((g_1 g_2)(x))$.

3.3

2. Give a big-O estimate for the number additions used in this segment of an algorithm.

```
t := 0
for i := 1 to n
    for j := 1 to n
        t := t + i + j
```

3. Give a big-O estimate for the number of operations, where an operation is a comparison or a multiplication, used in this segment of an algorithm (ignoring comparisons used to test the conditions in the for loops, where $a_1, a_2, \ldots, a_n$ are positive real numbers).

$$
\begin{aligned}
&m := 0 \\
&\text{for } i := 1 \text{ to } n \\
&\quad \text{for } j := i + 1 \text{ to } n \\
&\qquad m := \max(a_i a_j, m)
\end{aligned}
$$

43. Describe how the number of comparisons used in the worst case changes when these algorithms are used to search for an element of a list when the size of the list doubles from n to 2n, where n is a positive integer.
a) linear search
b) binary search

An $n \times n$ matrix is called upper triangular if $a_{ij} = 0$ whenever $i > j$.

45. From the definition of the matrix product, describe an algorithm in English for computing the product of two upper triangular matrices that ignores those products in the computation that are automatically equal to zero.

In Exercises 49 assume that the number of multiplications of entries used to multiply a p × q matrix and a q × r matrix is pqr.
49. What is the best order to form the product $ABCD$ if $A, B,$ C, and D are matrices with dimensions 30×10, 10×40, 40×50, and 50×30, respectively?

Chapter 3—Test 1

1. Describe an algorithm for finding the smallest integer in a finite sequence of integers.

2. Determine the worst case complexity in terms of the number of comparisons used for the algorithm you described in problem 1.

3. Let $f(n) = 3n^2 + 8n + 7$. Show that $f(n)$ is $O(n^2)$. Be sure to specify the values of the witnesses C and k.

4. Suppose that $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are 3×4, 4×5, and 5×6 matrices of numbers, respectively. Is it more efficient to compute the product $\boldsymbol{ABC}$ as $(\boldsymbol{AB})\boldsymbol{C}$ or as $\boldsymbol{A}(\boldsymbol{BC})$? Justify your answer by computing the number of multiplications of numbers needed each way.

Chapter 3—Test 2

1. (a) Describe an algorithm for finding the second largest integer in a sequence of distinct integers.

(b) Give a big-O estimate of the number of comparison used by your algorithm.

2. Show that $1^3 + 2^3 + 3^3 + \cdots + n^3$ is $O(n^4)$.

3. Show that the function $f(x) = (x+2)\log(x^2+1) + \log(x^3+1)$ is $O(x \log x)$.

4. Describe a brute-force algorithm for determining, given a compound proposition P in n variables, whether P is satisfiable. It is known that this problem is NP-complete. If $\mathrm{P} = \mathrm{NP}$, what conclusion can be drawn about the efficiency of your algorithm compared to the efficiency of the best algorithm for solving this problem?